Thermal-hydraulic Modeling and Simulation of Piston Pump

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Abstract: This paper presents a kind of modeling approach to the study of the thermal-hydraulic piston pump which is used in the airplane comprehensively. A set of lumped parameter mathematical models are developed which are based on conservation of energy. Heat transfer analysis for the piston pump is also given in the paper in which the heat flow inside the piston pump is described precisely. The theoretical basis and modeling strategy are applied in a typical thermal-hydraulic circuit containing the piston pump. Simulation results are presented which show a comparison of model/rig performance and the agreement obtained demonstrates the validity of the modeling approach.

Key words: thermal-hydraulic; piston pump; temperature; modelling

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摘　要：针对航空行业广泛应用的柱塞泵的热力学特性，提出了一种分析和建模的方法。应用能量守恒定律推导并建立了集中参数的航空柱塞泵的热力学模型，并针对柱塞泵的内部结构进行了较详细的传热分析。将此建模和分析的方法应用于包含航空柱塞泵的一个典型液压回路。通过对航空柱塞泵的热力学实验与仿真结果的比较证明了建模方法的有效性。

关键词：热液压；柱塞泵；温度；建模

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In hydraulic systems of an airplane, the oil temperature mainly depends on the efficiency of some working components, such as piston pumps, motors and valves. Often the piston pump is the main component in hydraulic power sources. As the developing of airborne hydraulic system, the output pressure and power of the piston pump are higher then ever. High output pressure and power will lead to larger power losses which can make the oil temperature become undesirably high. A highly elevated oil temperature can bring many problems to hydraulic systems and even make the system can not work properly. Therefore, it is necessary to make thermal hydraulic model of the piston pump and predict the temperature changes across the piston pump in the design process of the hydraulic power system.

Interest in the thermal consideration of piston pump has surfaced through years. In 1989, Kjølle[1] studied thermodynamic efficiency methods for pumps. In Norgard[2](1973) and Dorey and Harris[3] (1989) temperature calculations for pumps were discussed. These researches made it possible to predict the fluid temperatures of the thermal-hydraulic piston pump in the designing process. But these papers are mainly focus on the steady-state thermal calculation of the piston pump which can not give the thermodynamic model of the pump in detail.

In this paper, a generic and accurate thermal model of the piston pump is introduced and heat transfer analysis of the pump is presented. At the end of this work, the validity of the modeling approach is demonstrated by a comparison of model/rig performance of a hydraulic circuit including the piston pump.

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1 Basic Modelling Method

The conservation equation for energy is the basic equation for thermal-hydraulic modeling. For lumped-parameter models and one-dimensional flow, there is a simplified representation of this equation.

The basic relationship for the thermal change of the fluid over a hydraulic piston pump is the first law of thermodynamics applied to a flow process of the fluid volume[4]. The fluid volume is shown in Fig.1. For a complete and more comprehensive mathematical survey, see Refs. [5] and [6].

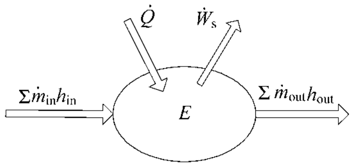

Fig.1 Control volume

The corresponding ordinary differential equation for fluid energy in a volume V is

$$\dot{Q}-\dot{W}=\sum\dot{m}_\mathrm{out}h_\mathrm{out}-\sum\dot{m}_\mathrm{in}h_\mathrm{in}+\dot{E} \tag{1}$$

where $\dot{W}$ is rate of work except the work required to push mass into and out of the control volume which is taken care of by using enthalpies for the energy of fluid streams. The energy E in the control volume is the sum of the internal energy U, the kinetic energy KE and the potential energy PE: $E=U+KE+PE$. If the kinetic and potential energy are neglected, the time rate of change of the energy can be expressed according to Eq.(2),

$$\frac{\mathrm{d}E}{\mathrm{d}t}=\frac{\mathrm{d}(mu)}{\mathrm{d}t}=m\frac{\mathrm{d}u}{\mathrm{d}t}+u\frac{\mathrm{d}m}{\mathrm{d}t} \tag{2}$$

Since the fluids in this study do not change phase, the specific enthalpy can be expressed as a function of temperature T and pressure p, that is $h=h(T, p)$. So the time derivative of h can be expressed as

$$\frac{\mathrm{d}h}{\mathrm{d}t}=\left(\frac{\partial h}{\partial T}\right)_p\frac{\mathrm{d}T}{\mathrm{d}t}+\left(\frac{\partial h}{\partial p}\right)_T\frac{\mathrm{d}p}{\mathrm{d}t} \tag{3}$$

The first term in Eq.(3) is recognized as the specific heat at constant pressure c_p,

$$c_p=\left(\frac{\partial h}{\partial T}\right)_p \tag{4}$$

The second term demands some rearranging using the so-called $T\mathrm{d}s$ equations[6]. It can be expressed as Eq.(5), where v is the specific volume,

$$\left(\frac{\partial h}{\partial p}\right)_T=v-T\left(\frac{\partial v}{\partial T}\right)_p=v-v\alpha_pT \tag{5}$$

Eq.(3) can be rewritten as Eq.(6) by Eqs.(4) and (5).

$$\frac{\mathrm{d}h}{\mathrm{d}t}=c_p\frac{\mathrm{d}T}{\mathrm{d}t}+(1-\alpha_pT)v\frac{\mathrm{d}p}{\mathrm{d}t} \tag{6}$$

The specific enthalpy is defined as

$$H=u+pv \tag{7}$$

Introducing Eqs.(6) and (7) in Eq.(2), Eq.(8) is obtained,

$$\frac{\mathrm{d}E}{\mathrm{d}t}=c_pm\frac{\mathrm{d}T}{\mathrm{d}t}-mT\alpha_pv\frac{\mathrm{d}p}{\mathrm{d}t}+h\frac{\mathrm{d}m}{\mathrm{d}t}-p\frac{\mathrm{d}V}{\mathrm{d}t} \tag{8}$$

The continuity equation for one-dimensional flow gives

$$\frac{\mathrm{d}m}{\mathrm{d}t}=\sum\dot{m}_\mathrm{in}-\sum\dot{m}_\mathrm{out} \tag{9}$$

Combining Eqs.(8), (9), (10) gives

$$\frac{\mathrm{d}T}{\mathrm{d}t}=\frac{1}{c_pm}\Big[\sum\dot{m}_\mathrm{in}(h_\mathrm{in}-h)+\sum\dot{m}_\mathrm{out}(h-h_\mathrm{out})+\dot{Q}-\dot{W}+p\frac{\mathrm{d}V}{\mathrm{d}t}+mT\alpha_pv\frac{\mathrm{d}p}{\mathrm{d}t}\Big] \tag{10}$$

In most thermal-hydraulic components, $\dot{W}$ represents the rates of boundary shaft and work work. It can be written as

$$\dot{W}=\dot{W}_\mathrm{s}+\dot{W}_\mathrm{b} \tag{11}$$

$\dot{W}_\mathrm{b}$ is the rate of boundary work. It is calculated by

$$\dot{W}_\mathrm{b}=p\frac{\mathrm{d}V}{\mathrm{d}t} \tag{12}$$

Introducing Eqs. (11) and (12) into Eq. (10) gives

$$\frac{\mathrm{d}T}{\mathrm{d}t}=\frac{1}{c_pm}\Big[\sum\dot{m}_\mathrm{in}(h_\mathrm{in}-h)+\sum\dot{m}_\mathrm{out}(h-h_\mathrm{out})+\dot{Q}-\dot{W}_\mathrm{s}+T\alpha_pV\frac{\mathrm{d}p}{\mathrm{d}t}\Big] \tag{13}$$

If it is assumed that the average enthalpy within the control volume equates to the leaving enthalpy regardless of the inlet conditions[7], Eq.(13) can be expressed as

$$\frac{\mathrm{d}T}{\mathrm{d}t}=\frac{1}{c_pm}\left[\sum\dot{m}_\mathrm{in}(h_\mathrm{in}-h)+\dot{Q}-W_\mathrm{s}+T\alpha_pV\frac{\mathrm{d}p}{\mathrm{d}t}\right] \tag{14}$$

The change in specific enthalpy within the control volume is related to the change in pressure and temperature by

$$h_\mathrm{in}-h=\bar{c}_p(T_\mathrm{in}-T)+(1-\bar{\alpha}_p\bar{T})\bar{v}(p_\mathrm{in}-p) \tag{15}$$

Eq.(15) has been derived by integrating fundamental relationships between enthalpy, pressure and temperature[7] and all fluid parameters are replaced with their mean values over the temperature and pressure range considered (indicated by barred notations). Eq.(13) or Eq.(14) is lumped parameter equation representing conservation of energy.

2 Heat Transfer Analysis

Usually, the variable displacement piston pump is used in the airplane. But the heat transfers between the variable and fixed piston pump are almost the same. So the schematic for the fixed displacement axial piston pump is represented in Fig.2 which is more intuitionistic than that for the variable piston pump. There are three fluid chambers in the pump.

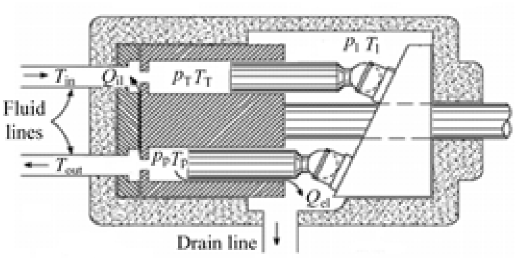

Fig.2 Fixed displacement axial piston pump

The model of thermal-hydraulic piston pump consists of four nodes: three fluid nodes and a mass node(Fig.3). The inlet fluid node represents the fluid volume in the inlet port; the outlet fluid node represents the fluid volume in the outlet port; the leakage node represents the fluid volume in the leakage port; the mass node represents the pump's wall. Usually, the materials for the wall have good conductivity. So heat conductance of the wall is neglected if the Biot number is less than approximately 0.1[8]. In this study, it is assumed that the Biot number of the component's wall is less than 0.1. The heat transfer

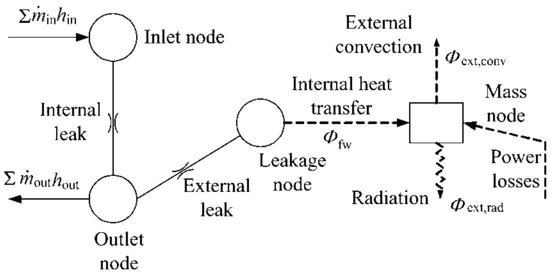

Fig.3 Sketch map of heat transfer

between the leakage node and the mass node is dominated by convection and the heat transfer between mass node and the environment is dominated by convection and radiation.

For the fluid/wall interface, the convective heat exchange Φ_{fw} is given by

$$\Phi_{fw} = k_{fw} A_{in} (T_f - T_w) \tag{16}$$

where k_{fw} is the fluid/wall heat transfer coefficient, A_{in} is the heat transfer area inside the wall, T_f is the fluid temperature and T_w is the wall temperature. The heat transfer coefficient k_{fw} is defined by

$$k_{fw} = \frac{\lambda Nu}{l} \tag{17}$$

where λ is the fluid conductance, l is the specific length, Nu is Nusselt number which is calculated by experiments. The functional expression for Nu depends on the respective component(see Ref.[9]).

For the wall/ambient interface, the heat transferred by radiation is given by

$$\Phi_{rad} = \varepsilon \sigma A_{wa} (T_w^4 - T_a^4) \tag{18}$$

where σ is Stefan-Boltzmann constant and ε is the emissivity. For ε typical values are taken from Refs. [10] and [11].

3 Mathematical Models

For the inlet and outlet fluid nodes, the time rate of change of the temperature can be expressed according to Eqs.(14) and (15).

$$\frac{dT_T}{dt} = \frac{1}{c_p m_p}\Big[\rho Q_{il}(c_p(T_p - T_T) + (1-\alpha_p(T_p + T_T)/2)\cdot$$
$$\nu(p_p - p_T)) + \rho\omega D(c_p(T_{in} - T_T)) + T_T \alpha_p V_T \frac{dp_T}{dt}\Big] \tag{19}$$

$$\frac{dT_p}{dt} = \frac{1}{c_p m_p}\Big[\rho\omega D(c_p(T_T - T_p) + (1-\alpha_p(T_p + T_T)/2)\cdot$$
$$\nu(p_T - p_p)) + T_p \alpha_p V_p \frac{dp_p}{dt}\Big] \tag{20}$$

where T_T is the temperature in the inlet fluid node, V_T is the volume in the inlet fluid node, T_p is the temperature in the outlet fluid node, V_p is the volume in the outlet fluid node, p_T is the pressure in the inlet fluid node, p_p is the pressure in the outlet fluid node, Q_{il} is the internal leakage flow rate, ω is the shaft rotational speed and D is the displacement of the pump.

The differential equations for the leakage node and mass node can be written as Eqs.(21) and (22) according to Eqs. (14), (15) and (16).

$$\frac{\mathrm{d}T_1}{\mathrm{d}t}=\frac{1}{c_p m_1}\Big[\rho Q_{\mathrm{el}}(c_p(T_{\mathrm{p}}-T_1)+(1-\alpha_p(T_{\mathrm{p}}+T_1)/2)\cdot \nu(p_{\mathrm{p}}-p_1))-k_{\mathrm{fw}}A_{\mathrm{fw}}(T_1-T_{\mathrm{w}})+T_1\alpha_p V_1\frac{\mathrm{d}p_1}{\mathrm{d}t}\Big] \tag{21}$$

$$\frac{\mathrm{d}T_{\mathrm{w}}}{\mathrm{d}t}=\frac{1}{c_{\mathrm{w}} m_{\mathrm{w}}}\Big[k_{\mathrm{fw}}A_{\mathrm{fw}}(T_1-T_{\mathrm{w}})-k_{\mathrm{wa}}A_{\mathrm{wa}}(T_{\mathrm{w}}-T_{\mathrm{a}})-\varepsilon\sigma A_{\mathrm{wa}}(T_{\mathrm{w}}^4-T_{\mathrm{a}}^4)+\omega D(p_{\mathrm{p}}-p_{\mathrm{T}})(1-\eta_{\mathrm{m}})/\eta_{\mathrm{m}}\Big] \tag{22}$$

where T_1 is the temperature in the leakage node, T_{w} is the temperature of the mass node, p_1 is the pressure in the leakage node, A_{wa} is the heat transfer area outside the wall, A_{fw} is the heat transfer area inside the wall, Q_{el} is the external leakage flow rate, η_{m} is mechanical efficiency of the pump and c_{w} is the specific heat of the wall. Eqs.(19), (20), (21) and (22) are the basic thermal equations to calculate the temperature changes in each chamber of the piston pump.

4 Simulation Results

The simulated system is shown in Fig.4. The system comprises a piston pump with external case drain (the type is YZB12), loading valve, heat exchanger and reservoir. Thermocouples can sensor the temperatures in the nodes of the pump. Although the hydraulic system is a little simple, it is enough to test the validity of the pump model. In the experiments, the system volume flow is changed by adjusting the load valve based on the real working conditions.

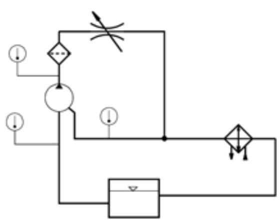

Fig. 4 Test rig for thermal-hydraulic analysis of the piston pump

In this study, the total test time is 8400 s. The volume flow change of the system is shown in Fig.5.

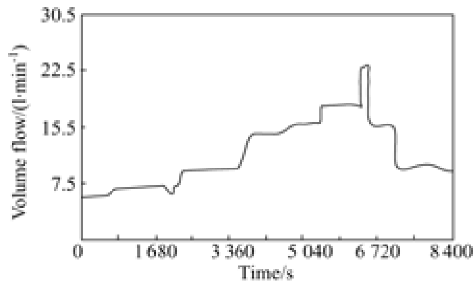

Fig. 5 Volume flow change of the system

Because it is assumed that the average enthalpy within the control volume equates to the leaving enthalpy, the fluid temperature leaving the volume is equal to the temperature in the volume. So the temperatures of outlet and leakage node are easy to measure. So in the test, the temperature of the outlet node T_{p}, leakage node T_1 and the temperature flow in to the pump T_{in} are measured. T_{in} is the input data according to Eq. (19).

The comparisons of the test and simulation temperatures of the pump's wall and leakage port are shown in Fig.6 and Fig.7. This comparison yields in a good agreement between calculated and measured temperatures.

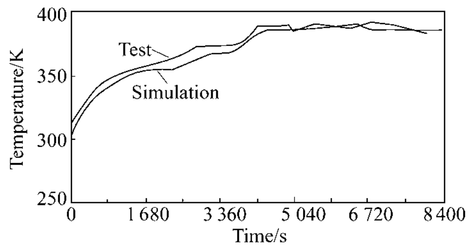

Fig. 6 Model/rig temperature response of pump's wall

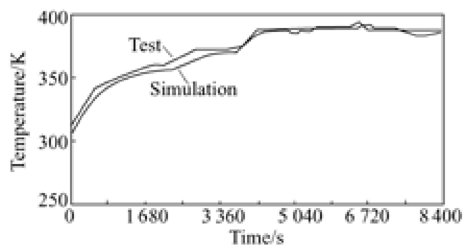

Fig. 7 Model/rig temperature response of pump's leakage port

5 Conclusions

The fundamental method of modeling thermal-hydraulic piston pump is introduced and the mathematical model of the pump is derived in this paper. The thermal-hydraulic model can be applied in any piston pump in thermal-hydraulic system calculations. For the circuit studied, the good correspondence is achieved between the model and rig. These results also validate the modelling methods.

Values of the thermal-hydraulic parameters for the piston pump are very important. So before calculating the temperature changes of the pump, the pump model and parameters must be validated through experiments.

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